Department of Systems Engineering & Operation Research
George Mason University
IT 735 / OR 735 Advanced Stochastic Simulation
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Benchmark Examples for Term Project

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Example 1. 10 Designs with Normal Distribution

$L(\theta_i, \xi) \sim N(i, 6^2)$, $i = 0, 1, .., 9$. We want to find a design with minimum expected mean. It is obvious that design 1 is the best.

Example 2. 10 Designs with Uniform Distribution

We consider a non-normal distribution for the performance measure: $L(\theta_i, \xi)) \sim \text{Uniform}(i\text{-}10.5, i+10.5)$, $i = 0, 1, .., 9$. We want to find a design with minimum expected mean. It is obvious that design 1 is the best.

Example 3. 10 Designs with Normal Distribution. Non-uniform Means and Unequal Variances.

$L(\theta_i, \xi) \sim N(mean, variance)$, $i = 0, 1, .., 9$. We want to find a design with minimum expected mean. This example also compares normally distributed designs, but the means and variances were (i.i.d.) randomly generated from Unif(0; 10) and Unif(24; 48) distributions, respectively, leading to the following values:

Means: 8.34, 0.98, 6.49, 0.10, 8.03, 6.21, 9.78, 9.10, 1.32, 3.27
Variances: 34.35, 44.34, 28.12, 35.93, 44.49, 43.39, 39.72, 24.31, 42.10, 24.42

Example 4. 100 Designs with Normal Distribution

To see how your algorithm perform in a bigger design space, $L(\theta_i, \xi) \sim N(i/10, 1^2)$, $i = 0, 1, 2, .., 98, 99$. We want to find a design with minimum expected mean. It is obvious that design 1 is the best.

Example 5. Worker Allocation Problem

As shown in Figure 1, this two-node tandem queueing system includes two stages of service. Suppose there is a total of 11 workers which will be allocated to these two stages. Customers arrive at the first node and leave the system after finishing the services at both stages. The service at each stage is performed by one worker. At least one worker must be allocated to each stage. When multiple workers are available in one stage, the services for multiple customers are performed in parallel and independently, i.e., the service of one customer will not become faster even if there are more workers than customers in a stage. However, customers have to wait if all workers in that stage are all busy. Further, the workers assigned to one stage can not help service at the other stage due to the training and safety requirement.

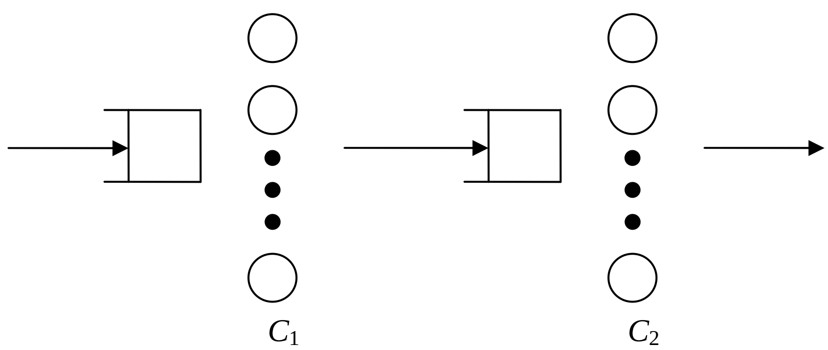


Figure 1. A two-node tandem queueing system with C_1 and C_2 workers at nodes 1 and 2, respectively.

The time to perform the service at stage 1 by one worker is uniformly distributed between 4 to 6 minutes, and the service time at stage 2 is uniformly distributed between 2 and 12 minutes. Customers arrive independently and the interarrival times between two customers are exponentially distributed with a rate of 1 customer per minute. To make customers happy, the manager of this service line wants the total time that a customer spends in the system as short as possible. A design question is how we should we allocate these 11 workers so that the average total time in system (also called system time) is minimized.

Denote C_1 and C_2 as the numbers of workers allocated to nodes 1 and 2. Thus $C_1 + C_2 = 11$, $C_1 \geq 1$, and $C_2 \geq 1$. There are 10 alternative combinations of (C_1, C_2). We want to choose the best alternative of (C_1, C_2) so that the average system for the first 100 customers is minimized. Since there is no close-form analytical solution for the estimation of the system time, stochastic simulation must be performed.

Example 6. (s,S) Inventory Control Problem

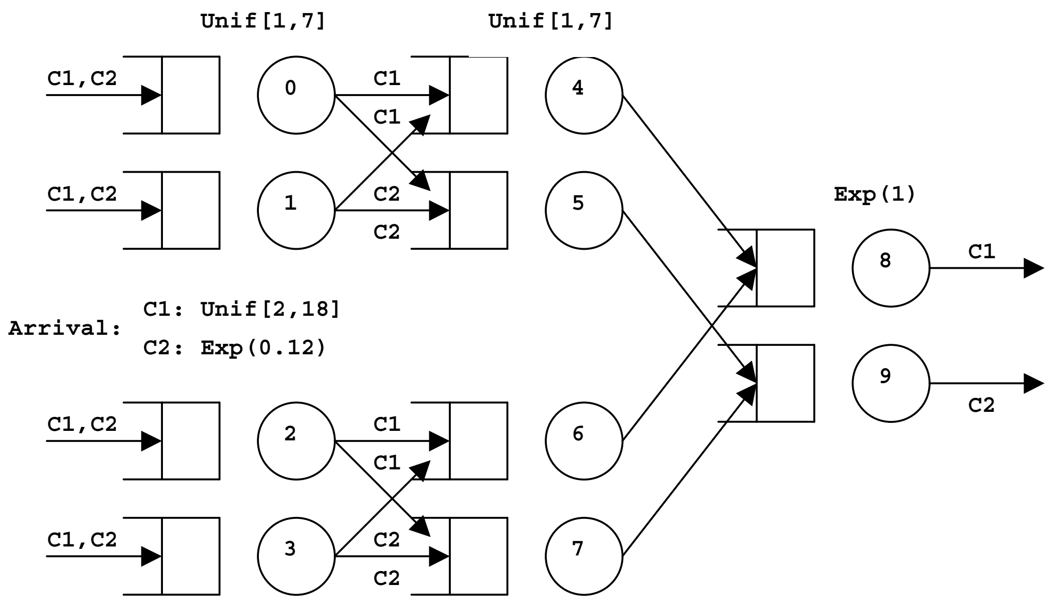
The second example is an (s, S) inventory policy problem based on the example given in Section 1.5.1 of Law and Kelton (2000). Recall that under an (s, S) inventory control policy, when the inventory position (which includes inventory on hand plus that on order) falls below s at an order decision point (discrete points in time in a periodic review setting and any point in time in a continuous review setting), then an order is placed in the amount that would bring the inventory position up to S.

Specifically, the system involves a single item under periodic review, full backlogging, and random lead times (uniformly distributed between 0.5 and 1.0 period), with costs for ordering (including a fixed set-up cost of \$32 per order and an incremental cost of \$3 per item), on-hand inventory (\$1 per item per period), and backlogging (fixed shortage cost of \$5 per item per period). The times between demands are i.i.d. exponential random variables with a mean of 0.1 period. The sizes of demands are i.i.d. random variables taking value 1, 2, 3, and 4, with probability 1/6, 1/3, 1/3, and 1/6, respectively.

The usual performance measure of interest involves costs assessed for excess inventory, inventory shortages, and item ordering. Alternatively, the problem can be formulated with costs on excess inventory and item ordering, subject to a service level constraint involving inventory shortages. Even for problem as simple as this one, there is no close-form expression to illustrate the inventory cost. Stochastic simulation is performed to estimate the such costs. A decision maker wants to find the optimal values of (s, S) in order to minimize the inventory cost.

Example 7. Buffer Allocation Problem in Communication Networks

We consider a 10-node network shown in the following figure. There are 10 servers and 10 buffers that are interconnected in a switching network.



We assume that there are two classes of customers with different arrival distributions, but the same service requirements. We consider both exponential and non-exponential distributions (uniform) in the network. Both classes arrive at any of Nodes 0~3, and leave the network after having gone through three different stages of service. Rather than probabilistic, the routing is class dependent as shown in Figure 1. Finite buffer sizes at all nodes are assumed which is exactly what makes our optimization problem interesting. Specifically, we are interested in distributing optimally a given number of buffer spaces to different nodes so that the network throughput is maximized. A buffer is said to be full if and when there are as many customers as its capacity allows, not including the customer being tended to in the server. We consider the problem of allocating 12 buffer units, among the 10 different nodes numbered from 0 to 9. We denote the buffer size of node *i* by θi. Specifically,

$$\theta_0 + \theta_1 + \theta_2 + \ldots + \theta_9 = 12, \text{ and } \theta_i \text{ is a non-negative integer.}$$

The number of different combinations of $[\theta_0, \theta_1, \theta_2, ..., \theta_9]$ which satisfy the above constraint can be calculated as follows:

$$\binom{12+10-1}{10-1} = 293{,}930.$$

Example 8. Continuous Design Space

$L(x, y, \xi) \sim f(x, y) + N(0, 1^2)$,

where $f(x, y) = (x_1 + x_2 + 1)^2 + 3x_2^3$

for $-2.0 \le x_1 \le 2.0$ and $-2.0 \le x_2 \le 2.0$.

In this example, we want to find x and y such that $E[\, L(x, y, \xi)\,]$ is minimized.

Example 9. Continuous Design Space

$L(x, y, \xi) \sim f(x, y) + N(0, 1^2)$,

where $f(x, y) = [\, 1 + (x_1 + x_2 + 1)^2\, (19 - 14x_1 + 3\, x_1^2 - 14\, x_2 + 6x_1x_2 + 3x_2^2)\,]$
$* [\, 30 + (2x_1 - 3x_2)^2\, (18 - 32x_1 + 12\, x_1^2 + 48\, x_2 - 36x_1x_2 + 27x_2^2)\,]$

for $-2.5 \le x_1 \le 2.0$ and $-2.5 \le x_2 \le 2.0$.

In this example, we want to find x and y such that $E[\, L(x, y, \xi)\,]$ is minimized.

More Examples.

You can find several well known examples in the literature. The following paper gives a nice description about 2 testing functions: (d) Griewank; and (b) Rosenbrock:

Chen, C. H., D. He, M. Fu, and L. H. Lee, "Efficient Simulation Budget Allocation for Selecting an Optimal Subset," *Informs Journal on Computing*, Vol. 20, No. 4, pp. 579-595, 2008.

Please check with Dr. Chen for the paper.