

Homework Assignment #1

due 7:20 pm, Monday, February 16

***** Due time will be strictly enforced. Late HW is subject to at least 25% penalty *****
***** There is a 20% penalty for the HW turned in through email *****

Definition: **Chen's Favorite Random Numbers** (CFRN) : { 0.025, 0.075, 0.125, 0.175, ..., 0.875, 0.925, 0.975 }. There are 20 numbers in this set. Obviously, these are not good random numbers. However, we will use these numbers several times in our assignments. For your convenience, there is an Excel file containing CFRN available at the class web site.

1. Consider the random variable X_i has an exponential distribution: $X_i \sim \text{EXP}(0.02)$.
 - (a) (5 points) Use CFRN to generate 20 random numbers for X. Give the 20 numbers { X_1, X_2, \dots, X_{20} } and their average and standard deviation.
 - (b) (5 points) Repeat part (a) using the function rand() in Excel, instead of using CFRN.

2. Suppose X is a discrete random variable with the following distribution:

$$P(X=1) = 0.1$$

$$P(X=2) = 0.5$$

$$P(X=3) = 0.25$$

$$P(X=4) = 0.1$$

$$P(X=5) = 0.05$$

- (a) (5 points) Use CFRN to generate 20 random numbers for X. Give the 20 numbers { X_1, X_2, \dots, X_{20} } and their average and standard deviation.
- (b) (5 points) Repeat part (a) using the function rand() in Excel, instead of using CFRN.

4. (10 points) Create an Alias Table for the discrete random variable X given in Question 2. Use CFRN to generate 10 random numbers for X . Give the 10 numbers $\{X_1, X_2, \dots, X_{10}\}$ and their average and standard deviation.

Note that you need two Unif(0,1) random number for each X , and so we can only generate 10 X 's using CFRN. Please use the first two numbers in CFRN to generate X_1 , use the 3rd and 4th numbers in CFRN to generate X_2 , and so on.

5. (20 points) Use Excel or any programming language to conduct the following simulation (without using CFRN)

- Step 1. Generate $U \sim \text{Unif}(0, 1)$
- Step 2. Generate $X \sim \text{Unif}(0, 1)$
- Step 3. If $U < 0.2$, do Step A
 Else if $(0.2 \leq U < 0.5)$, do Step B
 Else, do Step C
- Step A. $Y = X * 4$
- Step B. $Y = 1 + X * 2$
- Step C. $Y = 2 + X$

- (a) Repeat the above procedures to generate 5000 Y 's. Plot the histogram (relative frequency) for Y (no need to turn in these 500 numbers)
- (b) Give the theoretical probability distribution for Y . Hint: See Composition Method.

6. (10 points) Consider N_1 and N_2 as the decision variables. Find the optimal solution for the following maximization problem, in which $\mu_1 > \mu_2$.

$$\max_{N_1, N_2} \left(\frac{\mu_1 - \mu_2}{\sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}} \right), \text{ subject to } N_1 + N_2 = 10.$$