Department of Systems Engineering & Operation Research

George Mason University

IT 735/OR 735 Advanced Stochastic Simulation

Prof. Chun-Hung Chen Spring 2009

Homework Assignment #2

due 7:20 pm, Monday, April 6

Definition: **Chen's Favorite Random Numbers** (CFRN) : { 0.025, 0.075, 0.125, 0.175, ..., 0.875, 0.925, 0.975 }. There are 20 numbers in this set. Obviously, these are not good random numbers. However, we will use these numbers several times in our assignments. For your convenience, there is an Excel file containing CFRN available at the class web site.

1. (30 points) Apply Monte Carlo simulation to evaluate the integral

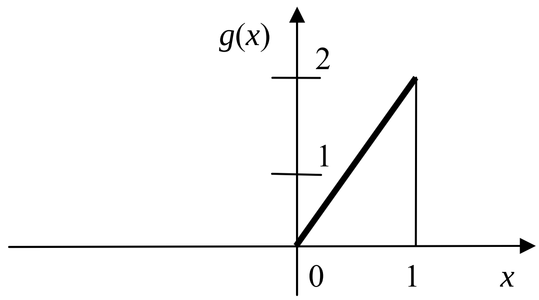
$$I = \int_0^1 x^4 dx .$$

Note that $I = E_x[x^4]$, where X is a random variable uniformly distributed between 0 and 1.

(a) Monte Carlo simulation: Take 100 samples of X. An unbiased estimator for I is $\frac{1}{100}\sum_{j=1}^{100} x_j^4$, where $x_j \sim$ Unif(0,1), i.i.d. Give your estimation.

(b) Continue part (a) but integrate antithetic variates into your simulation. Thus, x_j is an i.i.d. Unif(0,1) random number if j is an odd number, and $x_j = 1 - x_{j-1}$ if j is an even number. I is also estimated by $\frac{1}{100}\sum_{j=1}^{100} x_j^4$.

(c) Continue part (a) but integrate the importance sampling method. Since x^4 increases as x increases, it is a good idea to have higher sampling probability for higher x. So a good and simple choice of importance sampling distribution is a triangular distribution depicted below.



Derive your importance sampling estimator. Take 100 samples and give your estimation of *I*.

2. (20 points) For each of the three simulation methods in Question 1, replicate the procedure 20 times and calculate the mean and variance. Compare and discuss on the results you obtain.

3. (20 points) Suppose X is exponentially distributed with rate $\lambda = 1$.

(a) Generate 2,000 numbers for X using Excel or your own programming. Then use Arena Input Analyzer, or equivalent, to find a distribution with best fit. Please turn in a printout of your fitting report.

(b) Generate another random variate Y using the following the procedures:
Step 1. Generate a random number Z using the generator Exp(1.0) developed in part (a).
Step 2. If $Z < 5$, then return to Step 1 and repeat.
Step 3. $Y = Z - 5$.
Like part (a), generate 2000 numbers of Y (so you need more than 2000 numbers of Z). Then please use Arena Input Analyzer, or equivalent, to find a distribution with best fit. Please turn in a printout of your fitting report.

(c) Compare the results obtains in parts (a) and (b), and discuss.

4. (10 points) Apply Monte Carlo simulation and use the 20 numbers in CFRN to estimate the size of the area that $f(x,y) = x^2 + y^3 < 2$, for $0 \le x \le 2$ and $0 \le y \le 2$.. Specifically, please apply the 1st, 3rd, 5th, .., numbers to generate x, and the 2nd, 4th, 6th, ..., numbers to generate y.

5. (15 points) Continue Question 4.

(a) What is the 90% confidence interval if we apply the normal distribution model?

(b) What is the 95% confidence interval if we apply the t distribution model?

(c) Continue part (b), how many more Unif(0,1) numbers are needed if we want to reduce the length of confidence interval by 50%?

6. (15 point) Suppose there are 5 alternative designs under consideration. We want to simulate all the 5 designs and then to find the best design with minimum expected performance. Since the uncertainty of the system must be considered, stochastic simulation has to be run and multiple replications must be performed for each design. However, given the time constraint, we can only afford running a total of 50 simulation replications among these 5 designs. Based on some preliminary simulation and prior knowledge, the best guess we have for the means of the 5 designs are: 1, 2, 3, 4, and 5. The guess for the variances is: 1, 1, 9, 9, and 4. If we apply OCBA (Optimal Computing Budget Allocation), how should we allocate the total of 50 simulation replications to the 5 designs? To make your computation easier, you can assume that the number of simulation replications can be a floating number.

7. (10 point) Continue Question 6. If we allocate the simulation budget in a way that the number of replications is proportional to variance, how do you allocate the 50 simulation replications to the 5 designs? Such a way is similar with the very famous Rinott Procedure.