

Homework Assignment #3

due 4:30 pm, Wednesday, October 21

Definition: **Chen's Favorite Random Numbers** (CFRN) : { 0.025, 0.075, 0.125, 0.175, ..., 0.875, 0.925, 0.975 }. There are 20 numbers in this set. Obviously, these are not good random numbers. However, we will use these numbers several times in our assignments. For your convenience, there is an Excel file containing CFRN available at the class web site.

1. Suppose X is a continuous random variable which is uniformly distributed between (-4) and 6.

(a) (10 points) Use CFRN to generate 20 random numbers for X . Give the 20 numbers { X_1, X_2, \dots, X_{20} } and their average and standard deviation.

(b) (5 points) Repeat part (a) using the function rand() in Excel, instead of using CFRN.

2. Consider the random variable X_i which has an exponential distribution: $X_i \sim \text{EXP}(0.8)$.

(a) (10 points) Use CFRN to generate 20 random numbers for X . Give the 20 numbers { X_1, X_2, \dots, X_{20} } and their average and standard deviation.

(b) (5 points) Repeat part (a) using the function rand() in Excel, instead of using CFRN.

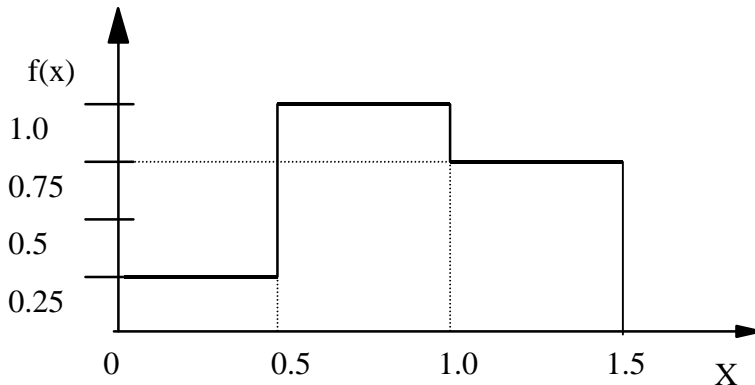
3. (10 points) Consider the cdf of a triangular distribution, $F(x)$, given in Chapter 6 of text book, which is

$$F(x) = \begin{cases} \frac{(x-a)^2}{(b-a)(c-a)}, & \text{if } a \leq x \leq c \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)}, & \text{if } c \leq x \leq b \end{cases}$$

Let $a=0.04$, $b=0.12$, and $c=0.08$. Give an Inverse Transform algorithm.

4. (25 points) Suppose X_i is normally distributed: $X_i \sim N(5, 2^2)$.
- (a) (10 points) Use CFRN to generate 20 X_i 's using the **original** Polar Method (in the original Polar Method, no random number will be rejected). Give the 20 numbers $\{X_1, X_2, \dots, X_{20}\}$ and their average and standard deviation. Note that two uniform numbers are needed for each call of the Polar Method. But you generate two normal numbers in each run. For example, you use the 1st and 2nd numbers to generate the first two normal numbers. Thus you will be able to generate 20 normal numbers using the 20 uniform random numbers.
- (b) (5 points) Repeat part (a) using the function rand() in Excel, instead of using CFRN.
- (c) (5 points) Apply the NORMINV function in Excel and use the 20 uniform numbers in CFRN to generate 20 random numbers for X. Give the 20 numbers $\{X_1, X_2, \dots, X_{20}\}$ and their average and standard deviation.
- (d) (5 points) Repeat part (c) using the function rand() in Excel, instead of using CFRN.

5. Suppose X has the following probability density function:

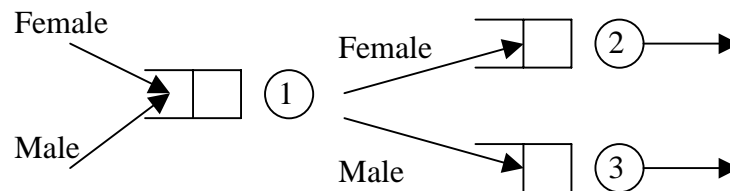


- (a) (10 points) Use CFRN to generate X_i 's using Acceptance-Rejection Method. Let $d = 1.0$, $a = 0$, and $b = 1.5$. Note that two uniform numbers are needed in one iteration of Acceptance-Rejection Method. To obtain a unique solution, please use the 1st and 2nd numbers in CFRN for the 1st iteration, use 3rd and 4th numbers in CFRN for the 2nd iteration, and so on. Thus you will be able to perform the A-R method for 10 iterations and generate no more than 10 Xs using these 20 uniform random numbers. Give the numbers you accept and calculate their average and standard deviation.
- (b) (5 points) Repeat part (a) using the function rand() in Excel, instead of using CFRN.

*** For Questions 6 and 7,

- TA and the instructor will answer only questions about problem clarification before HW due date
- In addition to the printout requested below, please turn in a floppy disk or CD-ROM containing all of your Arena models

6. Apply Arena for the following three-node system. There are two types of arriving customers: Male and Female. The inter-arrival times for Male customers are exponentially distributed with rate 0.5 (so the mean is 2.0), and the inter-arrival times for Female customers are also exponentially distributed with rate 0.5. After arriving at the system, some preliminary process has to be performed at Node 1. There is one worker to perform this work. The service time is Triangular(0.5, 0.8, 1.5). After the service at Node 1, Female customers are sent to Node 2 and Male customers are sent to Node 3 for further service. There is one worker at Node 2 and another worker at Node 3 to perform the services. The service times at Node 2 are Uniform(0.5, 3.0), and the service times at Node 3 are also Uniform(0.5, 3.0).



Have the following simulation setup:

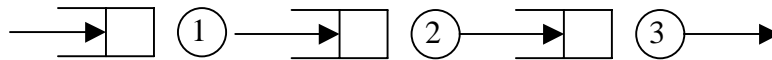
Number of Replications: **1**
Length of Replication **5000**
Beginning Time: **0**

(a) (10 points) Print your Arena model and a single page of your output report showing the average system times for Male and Female customers (please mark these two numbers).

(b) (10 points) Repeat Part (a). But we want to make a change about where Female customers are sent after the service at Node 1. Each Female customers will check the length of the queue at Node 2 to decide to where to go next. If the queue length of Node 2 is 0 or 1, this Female customer will go to Node 2. Otherwise, she goes to Node 3 instead.

(c) (10 points) Consider a different scenario from Part (a). Suppose we hire an additional worker (say number 4) to help the further service for Female customers. After the preliminary process at Node 1, each Female customer check the queue length of Node 2. If the queue length is 0 or 1, this Female customer will go to Node 2. Otherwise, she goes to Server 4 instead. Assume the service time by Server 4 is the same as that at Node 2, i.e., Uniform(0.5, 3.0).

7. Consider the following three-stage service system:



Each customer will be served by a SAME worker through all 3 stages. There are 5 workers to provide service. Suppose the interarrival times are exponentially distributed with **rate** 0.8 (the mean is 1.25), the service time distributions are:

Stage 1: Unif[0.5, 1.5),

Stage 2: Triangular(0.5, 1, 1.5)

Stage 3: Unif[0.6, 1.4)

Use ARENA to simulate this system. Have the following simulation setup:

Number of Replications:	1
Length of Replication	5000
Beginning Time:	0

(a) (10 points) Print your Arena model and a single page of your output report showing the average system times (please mark this number).

(b) (5 points) Repeat Part (a). Suppose there are only 4 workers to perform the service.

(c) (10 points) Continue Part (b) for a different scenario that each customer will be served by a SAME worker at Stages 1 and 2 only. At Stage 3, the service will be performed by a different worker. Now we allocate 3 workers to provide service at Nodes 1 and 2. And there is only one worker to perform service at Node 3.