

Homework Assignment #5

due 1:30 PM, Monday, April 27

****** Due time 1:30 PM will be strictly enforced. Late HW is subject to at least 25% penalty. To avoid late penalty, you are welcome to turn in your HW earlier than the due date.***

Definition: **Chen's Favorite Random Numbers** (CFRN) : { 0.025, 0.075, 0.125, 0.175, ..., 0.875, 0.925, 0.975 }. There are 20 numbers in this set. Obviously, these are not good random numbers. However, we will use these numbers several times in our assignments. For your convenience, there is an Excel file containing CFRN available at the class web site.

1. (10 points) Use the Arena Input Analyzer to find the best fitted distribution for the following two sets of data which are available at the class web site. Preserve all default Arena settings. Please hand in a printout of the distribution summary.

- (a) Group A data in HW#2. There are 100 data points, each is a sample of the investment annual return rate.
- (b) Group B data in HW#2. Also, there are 100 data points, each is a sample of the investment annual return rate.

2. (10 points) Read Chapter 5 of the Arena Book and then run the following two Arena examples (usually located at C:\Program Files\Rockwell Software\Arena\Arena Book):

- (a) Mod_05_1.doe (the example presented in section 5.1-5.6 of the Arena book)
- (b) Mod_05_2.doe (the example presented in sections 5.7-5.8 of the Arena book)

Please note that the section numbers may vary if you use different version of Arena book. The bottom line is that you are requested to read the entire chapter 5. Please turn in a printout of the first page of the "Category by Replication" report for each example.

3. In this question, we want to develop a simulator for planning your investment. A similar simulator developed by T. Rowe Price can be accessed via our class web site. Consider the following two options to invest your money:

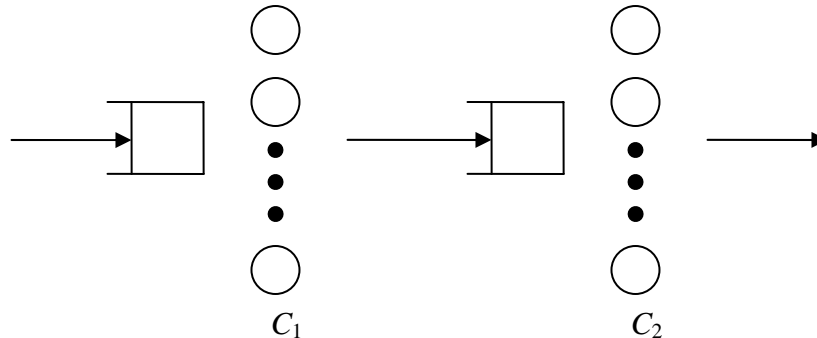
I. CD: low return rate without any risk.

II. Stock: high return rate with high risk

Based on historical data analysis, we have found that the yearly return rate for CD is 6%. Due to the risk of stocks, the yearly return rates are not guaranteed, but can be represented by statistical distribution: Uniform(-40%, 60%). This means that you may lose your money if you buy stocks, although the return rate can be as high as 60% per year. Suppose you have \$1,000,000 for investment. Develop a Monte Carlo simulator to evaluate the investment for a 4-year period.

- (a) (6 points) What is the expected total after 4 years if you invest all of your money in CD?
- (b) (12 points) For stock, use CFRN to conduct 5 simulation runs. In each run, please plot a curve which indicates the total money you will have at the end of each year. Your end result will be 5 curves something like the picture shown the above T. Rowe Price web site. In this case, you need four Unif(0,1) random numbers in one run. Use the 1st, 2nd, 3rd, and 4th numbers (i.e., 0.025, 0.075, 0.125, 0.175) to generate the four random numbers for first run. Then you use 5th~8th numbers for next run, etc. Thus you will be able to have 5 simulation runs using the 20 CFRN numbers. In addition, estimate the expected total and estimate the probability that your total is less than \$500,000 at the end of the 4-year period, if you invest all of your money in stocks.
- (c) (12 points) Repeat part (b). Instead of using CFRN, use the build-in function rand() in Excel and have 100 simulation runs (so you need 400 random numbers). Specifically, You have to i) Plot the 100 curves which indicates the total money you will have over the period of 4 years for the 100 simulation runs; ii) Estimate the expected total if you invest all of your money in stocks; and iii) Estimate the probability that your expected total is less than \$500,000 at the end of the 4-year period.

4. (30 points) Consider a two-node tandem queueing system shown in the following figure, which has two stages of service.



Suppose there is a total of 11 workers which will be allocated to these two stages. Customers arrive at the first node and leave the system after finishing the services at both stages. The service at each stage is performed by one worker. At least one worker must be allocated to each stage. When multiple workers are available in one stage, the services for multiple customers are performed in parallel and independently, i.e., the service of one customer will not become faster even if there are more workers than customers in a stage. However, customers have to wait if all workers in that stage are all busy. Further, the workers assigned to one stage can not help service at the other stage due to the training and safety requirement.

The time to perform the service at stage 1 by one worker is uniformly distributed between 4 to 6 minutes, and the service time at stage 2 is uniformly distributed between 2 and 12 minutes. Customers arrive independently and the interarrival times between two customers are exponentially distributed with a rate of 1 customer per minute. To make customers happy, the manager of this service line wants the total time that a customer spends in the system as short as possible. A design question is how we should we allocate these 11 workers so that the average total time in system (also called system time) is minimized.

Denote C_1 and C_2 as the numbers of workers allocated to nodes 1 and 2. Thus $C_1 + C_2 = 11$. We want to find the best alternative of (C_1, C_2) so that the average system time for the first 200 customers is minimized. Consider the following three alternatives:

- (a) $C_1 = 4$, and $C_2 = 7$.
- (b) $C_1 = 6$, and $C_2 = 5$.
- (c) $C_1 = 8$, and $C_2 = 3$.

Create an Arena model for this problem. For each alternative design, run your Arena simulation and turn in (only) the page of the report showing the average system time for the first 200 customers. Also please turn in a printout of your Arena model.

5. (15 points) Arena collects several useful model examples in the "SMART Files Library". Those models demonstrate a variety of modeling techniques and situations commonly encountered using Arena. They are very helpful for your model-building efforts. Go to "Arena Online Help" to read through the instructions about "SMART Files Library". For each of the following smart models, run the simulation and investigate what the model is demonstrating. Give one to two sentences of summary for each.

(a) Smarts042

(b) Smarts071

(c) Smarts077