Very Efficient Simulation for Decision Making under Uncertainty

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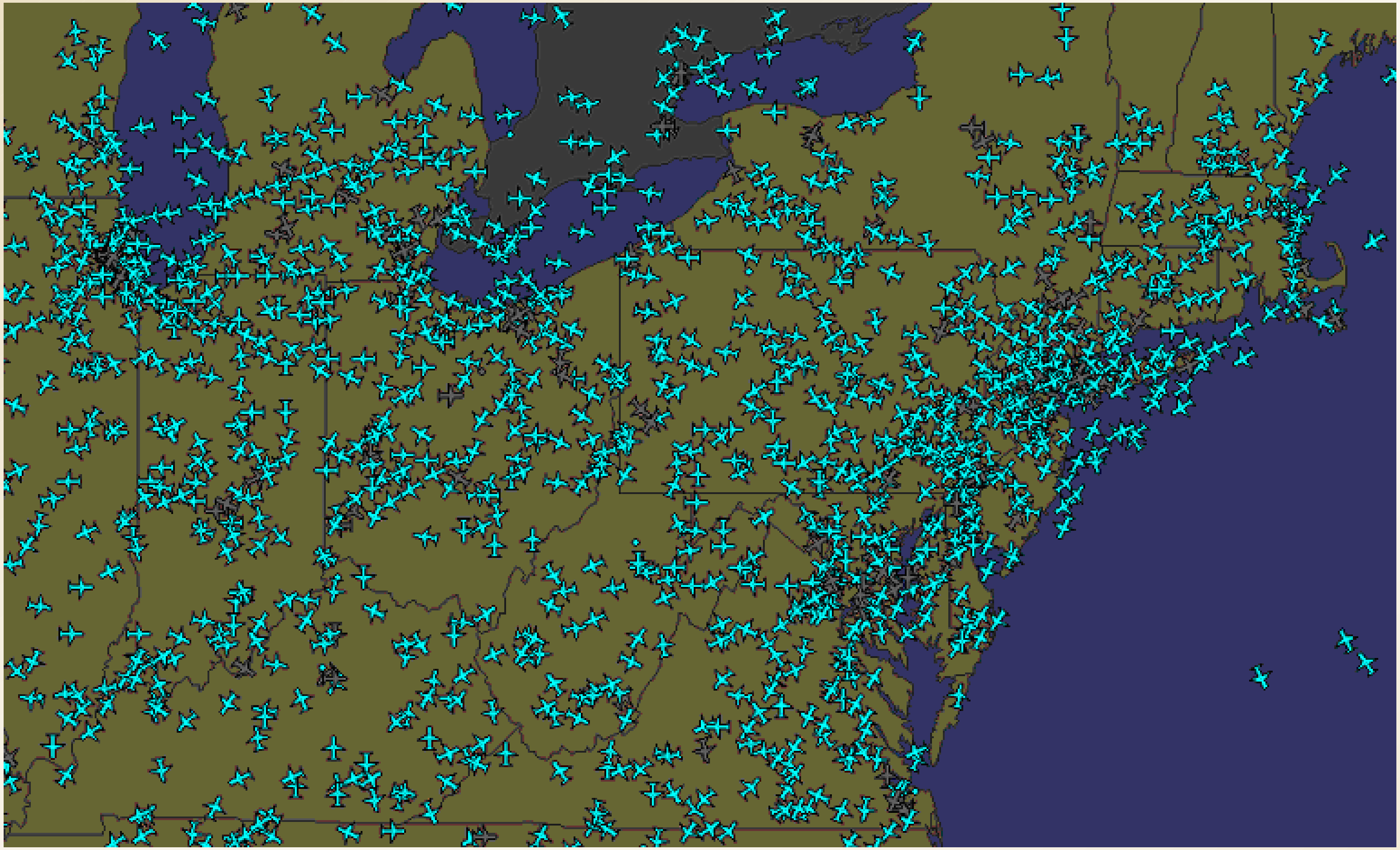
Outline

- Motivations
- Simulation-based optimization under uncertainty
- Very efficient simulation sampling technique:

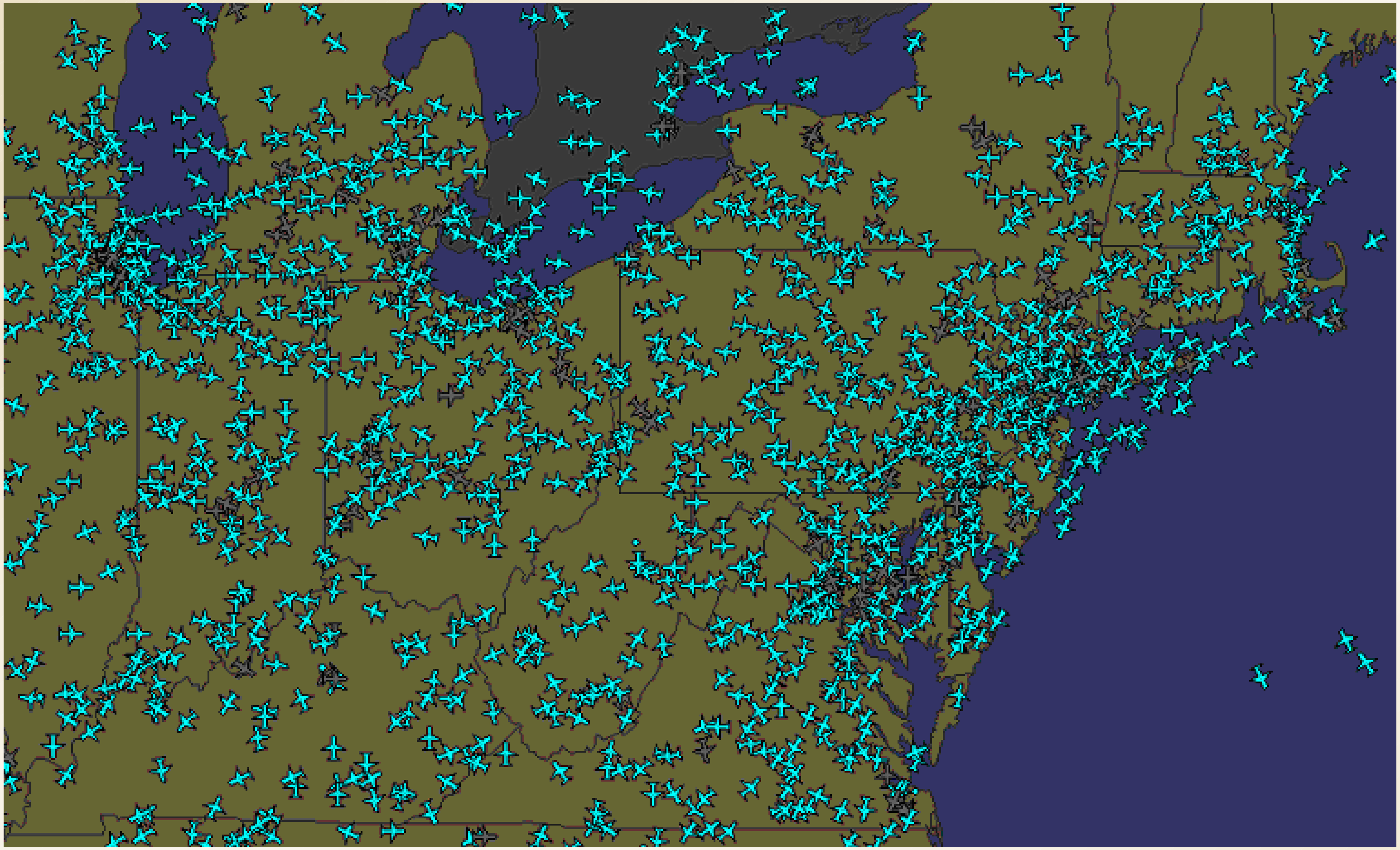
  Optimal Computing Budget Allocation (OCBA)
- Numerical testing
- Extension to more general optimization problems

  ⇒ Focus of this class

  ⇒ Term project



Why Simulation?



Many Advantages for Simulation-based Approach:

- Model complex and large systems
- Handle logical relationships, complex system dynamics, and precedence-based decision rules (e.g., discrete-event simulation)
- Consider uncertainty, risk, and variation (via Monte Carlo sampling)
- Provide excellent dynamic & interactive animation
- Answer what-if questions

Engineering Design

A Generic Example (Optimization Under Uncertainty)

Maximize $J(X) = E_W[\ U(\ f(X, W)\ )\ ]$

subject to system constrains and/or limited resources

where $U(\cdot)$ reflects the utility/risk attitude of the decision maker

f is a vector of system performance measures

X is the vector of decision/control variables

W represents the uncertainty/variation

Assume $X \in [\ X_1, X_2, X_3, .., X_k\ ]$,

k is the number of alternative designs (e.g., 1000)

Simulation as An Engineering Design Tool

$J(X)$ is estimated using Monte Carlo simulation or discrete-event simulation

Many Advantages:

- Model complex and large systems
- Handle logical relationships, complex system dynamics, and precedence-based decision rules (e.g., discrete-event simulation)
- Consider uncertainty, risk, and variation (via Monte Carlo sampling)
- Provide excellent dynamic & interactive animation
- Answer what-if questions

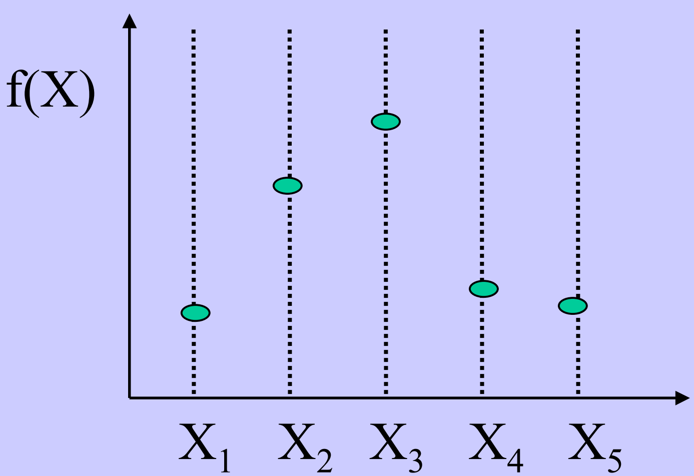
Disadvantage:

Efficiency is still a big concern

Function Evaluation: Deterministic vs. Stochastic

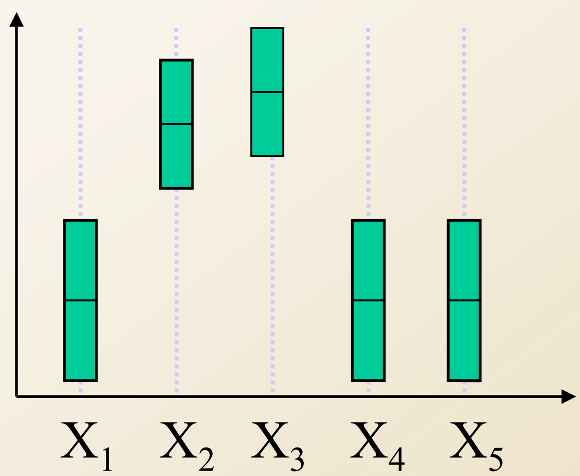
Deterministic:

an exact value f(X)



Stochastic:

a confidence interval (say 99%) obtained from multiple runs/samples/ replications/evaluations



Precision of Simulation Estimator

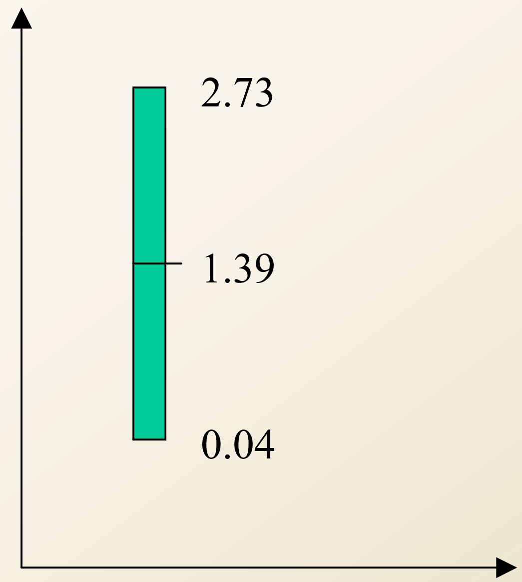
Confidence Interval (C.I.)

$$\frac{1}{N}\sum_{j=1}^{N} u(f(x_i, w_{ij})) \pm \frac{z\sigma}{\sqrt{N}}$$

where

- z is the critical value for the standard normal distribution
- σ is the standard deviation of one simulation sample
- N is the number of simulation runs (samples)

99% Confidence Interval

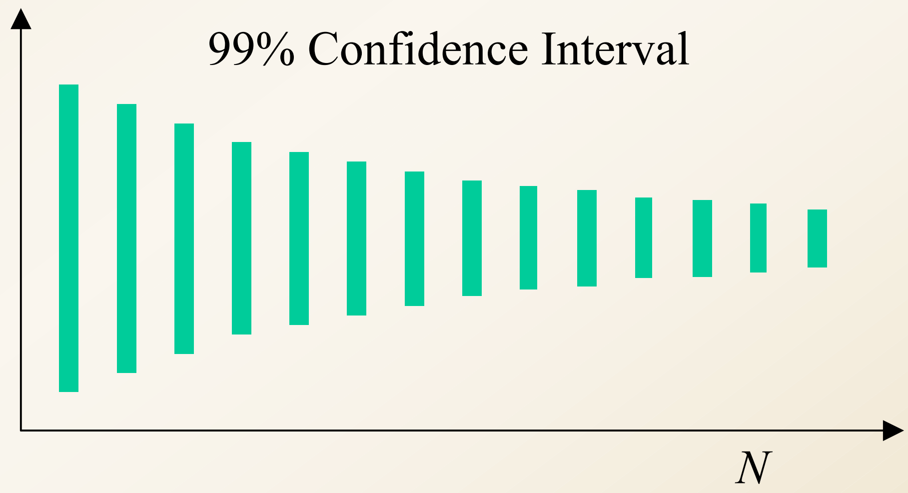


Estimation Uncertainty Reduction

Confidence Interval

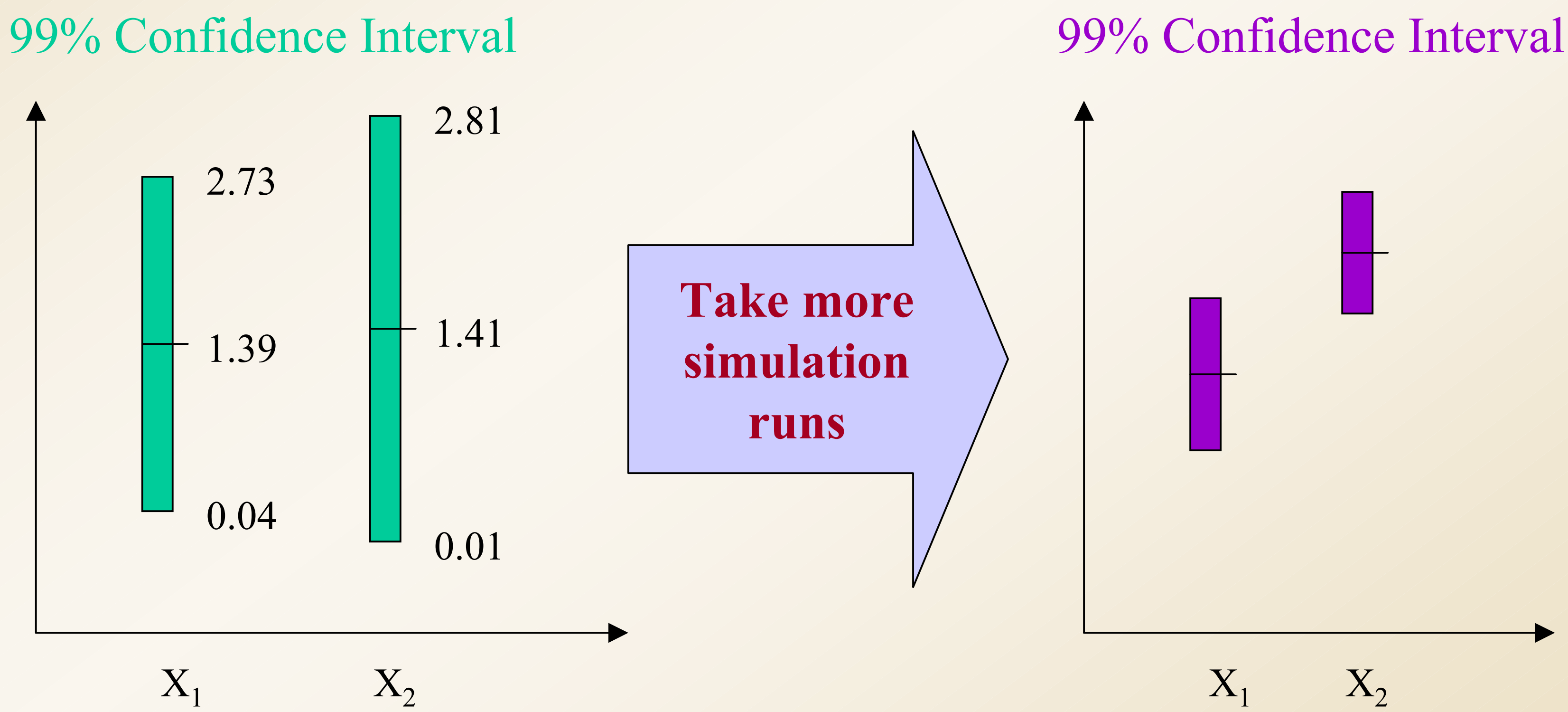
$$\frac{1}{N}\sum_{j=1}^{N} u(\,f(x_i,\,w_{ij})\,)\ \pm\ \frac{z\sigma}{\sqrt{N}}$$

Increase the number of simulation runs (*N*)



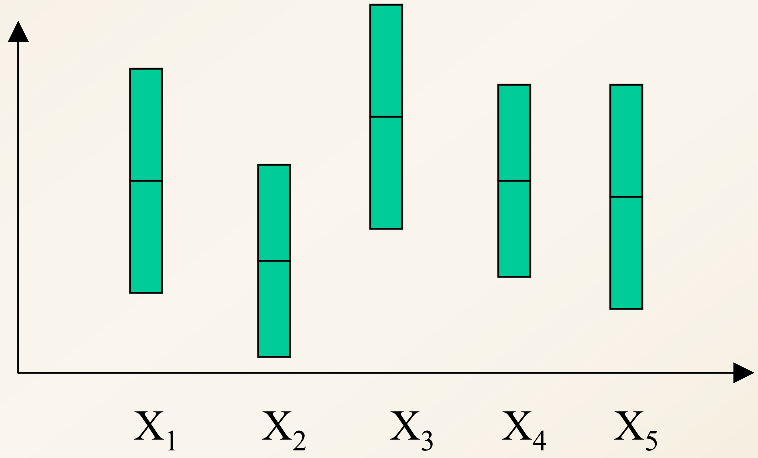
Comparison of Two Alternative Designs

Which decision has higher expected utility?



Multiple Comparison in Design Problems

Multiple Comparison of *k* alternative designs with estimation uncertainty



Correct Selection Probability

P\{CS\} = P\{Correct Selection of the True Best Design\}

becomes bigger as we increase N

Simulation efficiency is a big concern

– kN could be large → **time-consuming**

Simulation Methodologies

Traditional Methods

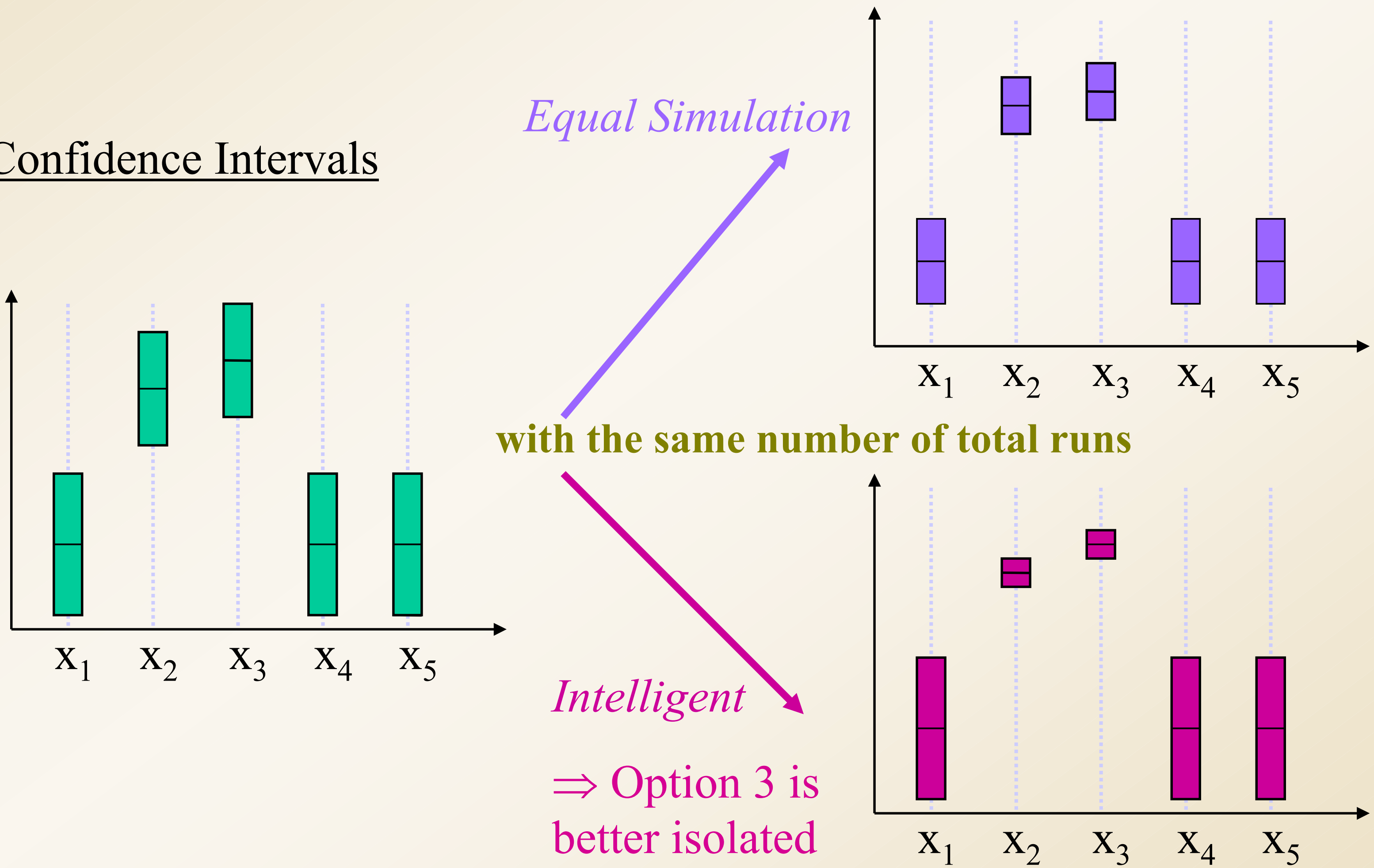
- Equal Simulation
  - All designs are equally simulated
- Rinott & Dudewicz Procedures: well-known procedures in OR
  - Proportional to variance
  - $N_i = c_i \sigma_i^2$
- Importance Sampling
  - Depends on the particular model of interest
  - Only the information of each design is used to reduce its variance for improving simulation efficiency

Our Intelligent Simulation

Optimal Computing Budget Allocation (OCBA)

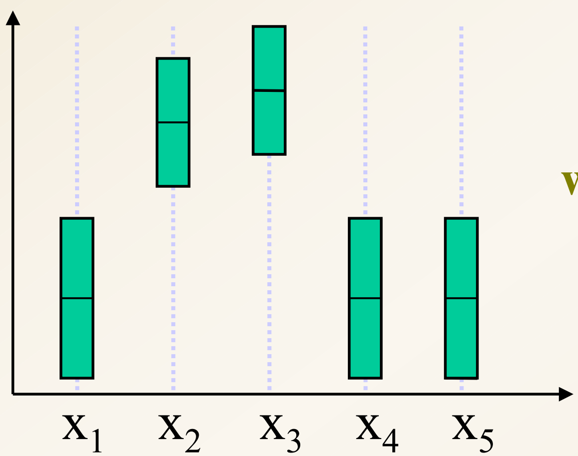
An Intuitive Example - Maximization

99% Confidence Intervals

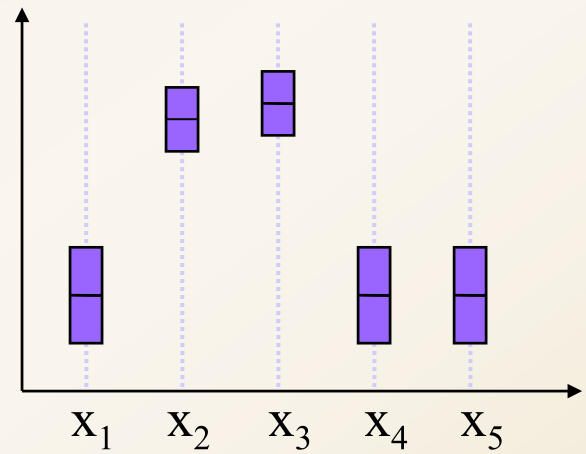


Marriage Problem

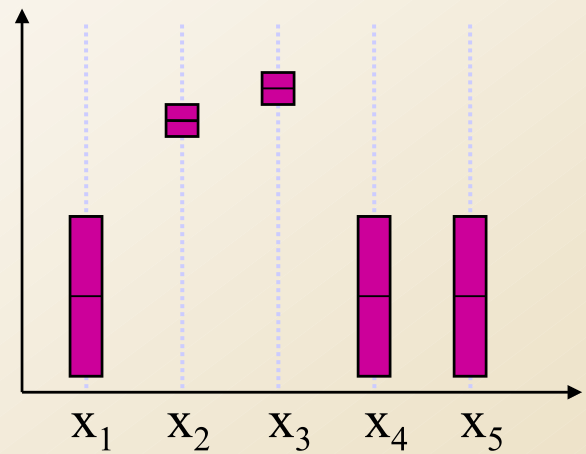
- 5 boys/girls to date
- Maximize your marriage utility function



Equal Dating



with the same number of total dates

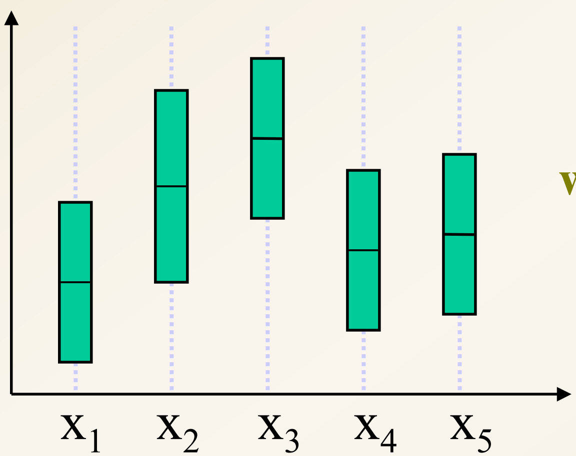


Intelligent Dating

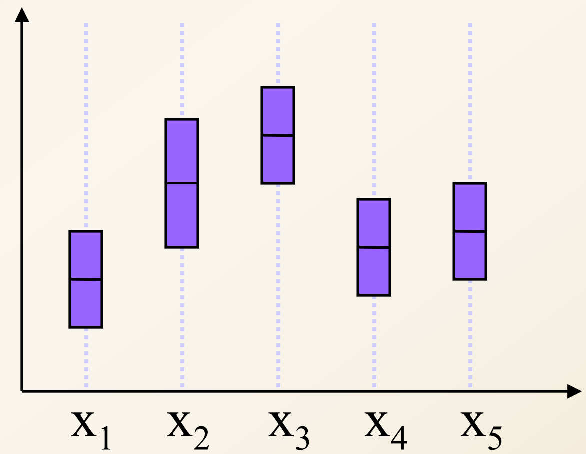
⇒ # 3 is standing out

More General Case

99% Confidence Intervals



Equal Simulation



with the same number of total runs

Intelligent
⇒ Option 3 is better isolated

Optimal Computing Budget Allocation (OCBA)

(P1) ***Minimize the total number of simulation runs in order to achieve a desired simulation quality:***

$$\min_{N_1,..,N_k} \; [\, N_1 + N_2 + .. + N_k \,]$$

$$\text{s.t. } P\{\text{CS}\} > \text{P}_{\text{sat}} \;\; \text{(a satisfactory level)}$$

(P2) ***Maximize the simulation quality with a given simulation budget:***

$$\max_{N_1,..,N_k} \; P\{\text{CS}\}$$

$$\text{s.t. } N_1 + N_2 + .. + N_k = T \;\; \text{(total comp. budget)}$$

⇨ **An *optimal* way to simulate alternative options for finding the *best* decision**

Asymptotic Solution of OCBA

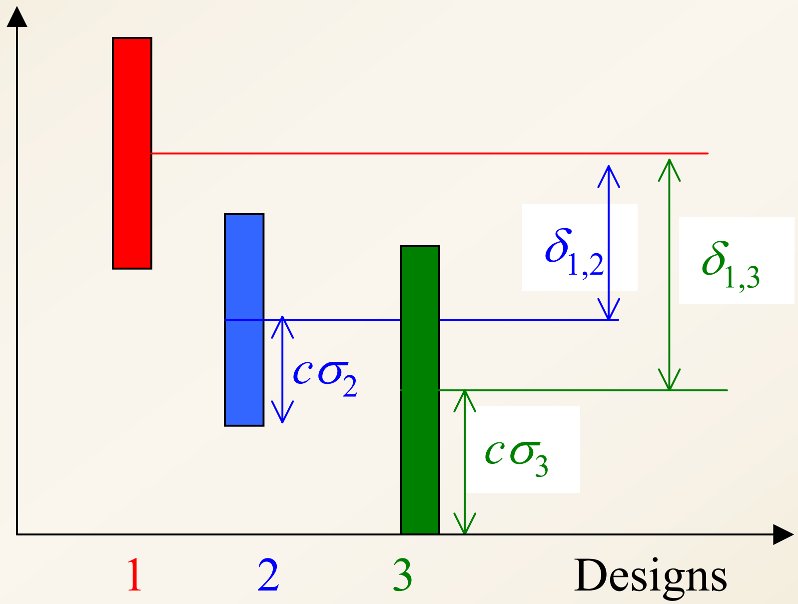
Given a total number of simulation runs T to be allocated to k competing designs, as $T \to \infty$, the $P\{\text{CS}\}$ can be asymptotically maximized when

★ $$\frac{N_i}{N_j} = \frac{(\sigma_i / \delta_{b,i})^2}{(\sigma_j / \delta_{b,j})^2} \quad \text{for } i \neq j \neq b$$

★ $$N_b = \sigma_b \sqrt{\sum_{i \neq b} (N_i^2 / \sigma_i^2)}$$

Intuitive Explanations

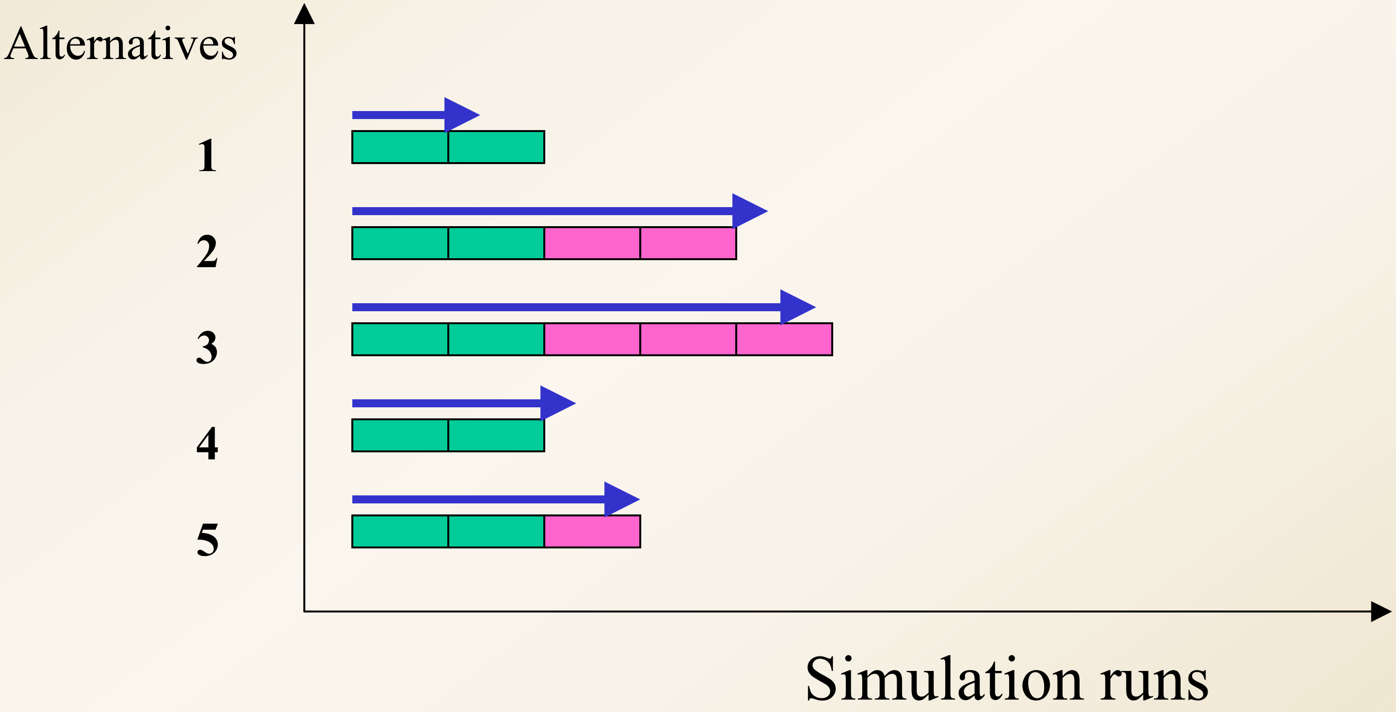
$$\frac{N_2}{N_3} = \frac{\sigma_2^2 \delta_{1,3}^2}{\sigma_3^2 \delta_{1,2}^2}$$



$\Rightarrow$ **N_2 increases when σ_2 or $\delta_{1,3}$ increases**

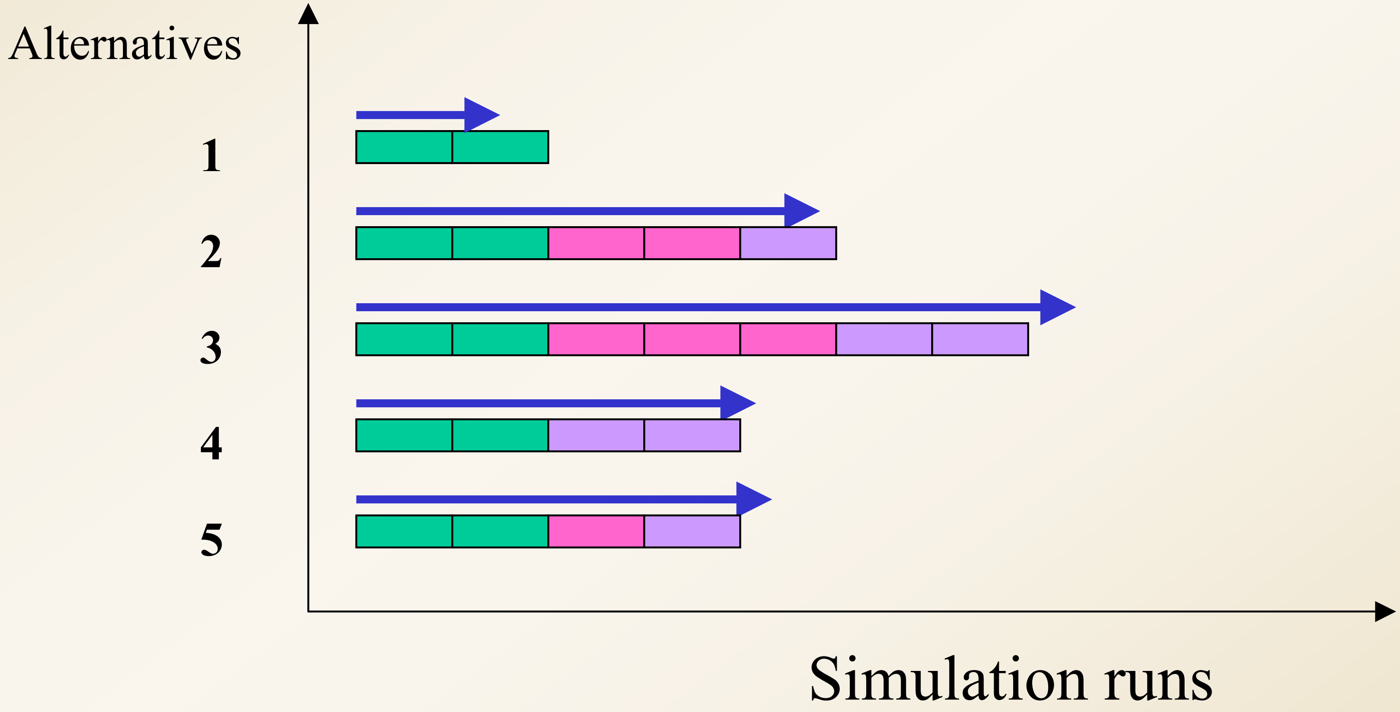
$\Rightarrow$ **N_2 decreases when σ_3 or $\delta_{1,2}$ increases**

Simulation Run Allocation Using OCBA



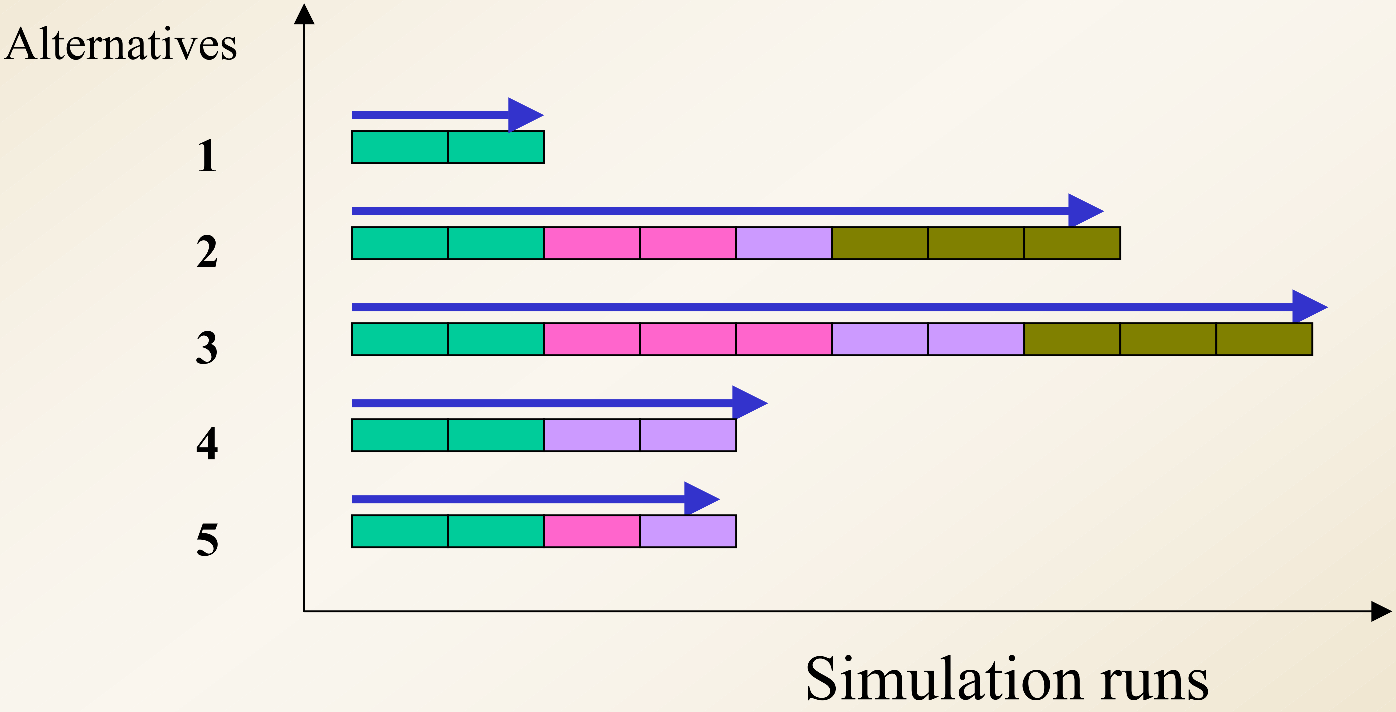
Q. If T is increased to 16?

Simulation Run Allocation Using OCBA



Q. If T is increased to 22?

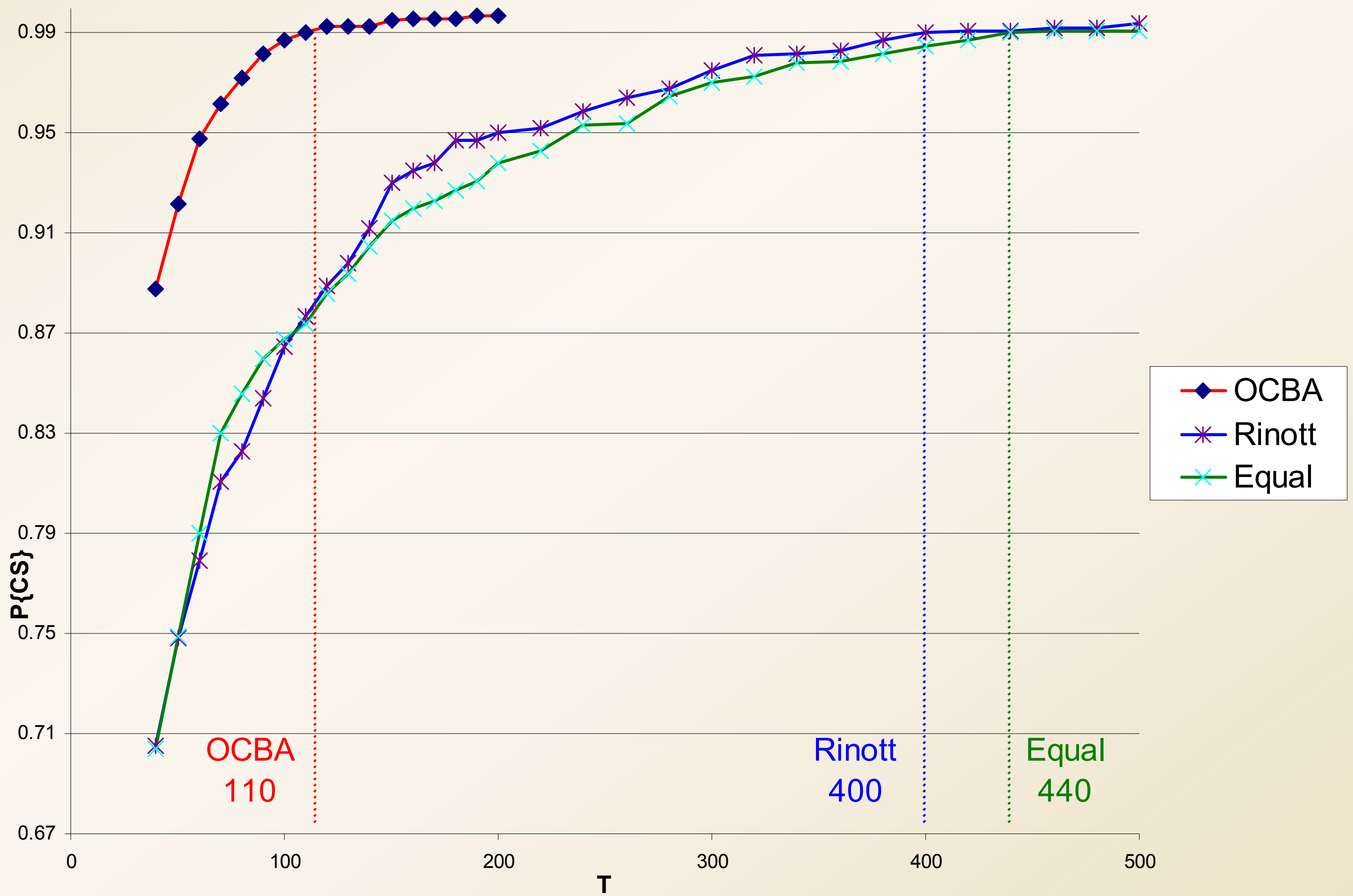
Intelligent Simulation Using OCBA



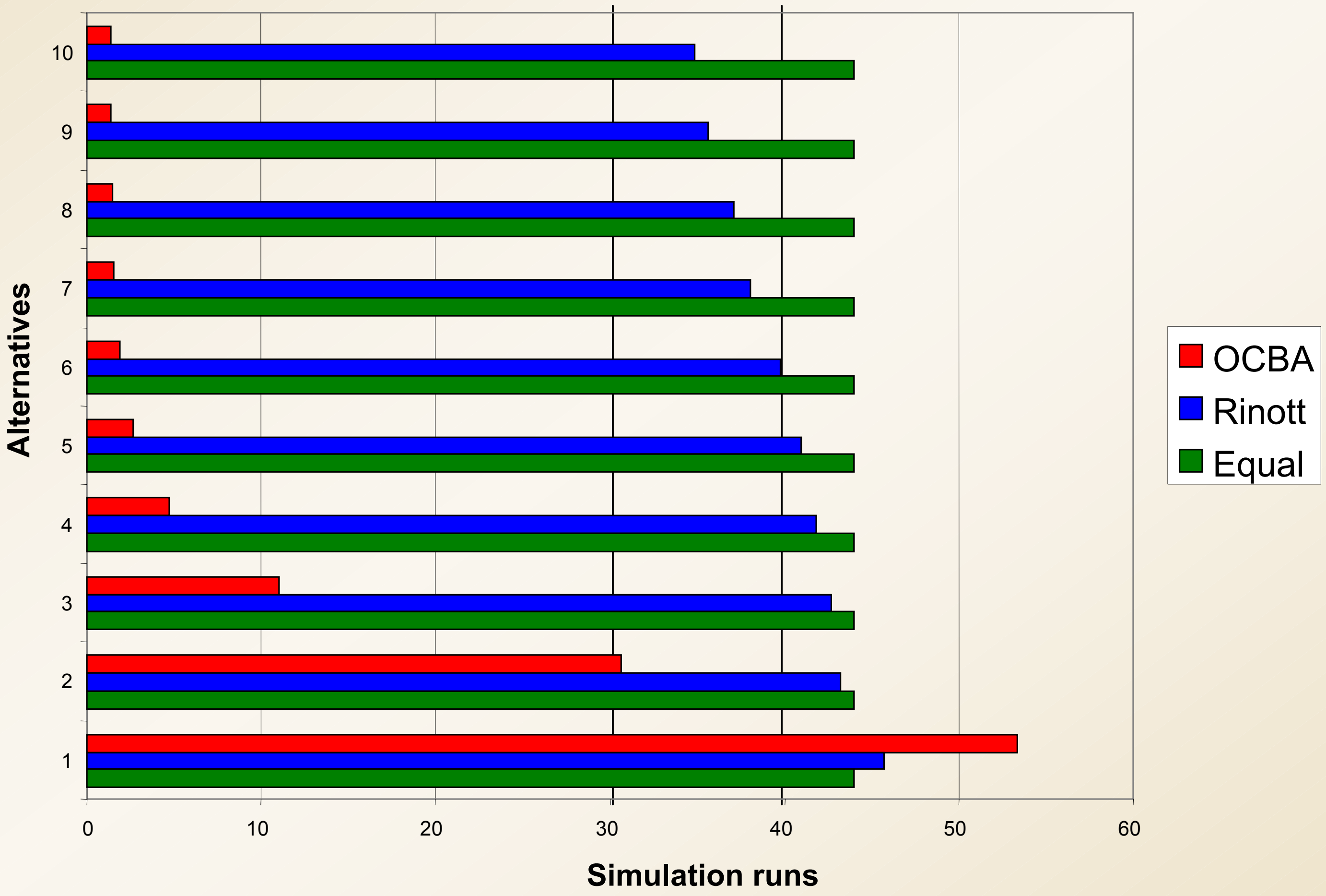
Q. If T is increased to 28?

Numerical Testing - 10 Alternatives

- $P\{\text{CS}\} \nearrow$ as the computing budget $T \nearrow$
- OCBA is more than 3 times faster in achieving 99% of $P\{\text{CS}\}$



Simulation Run Allocation

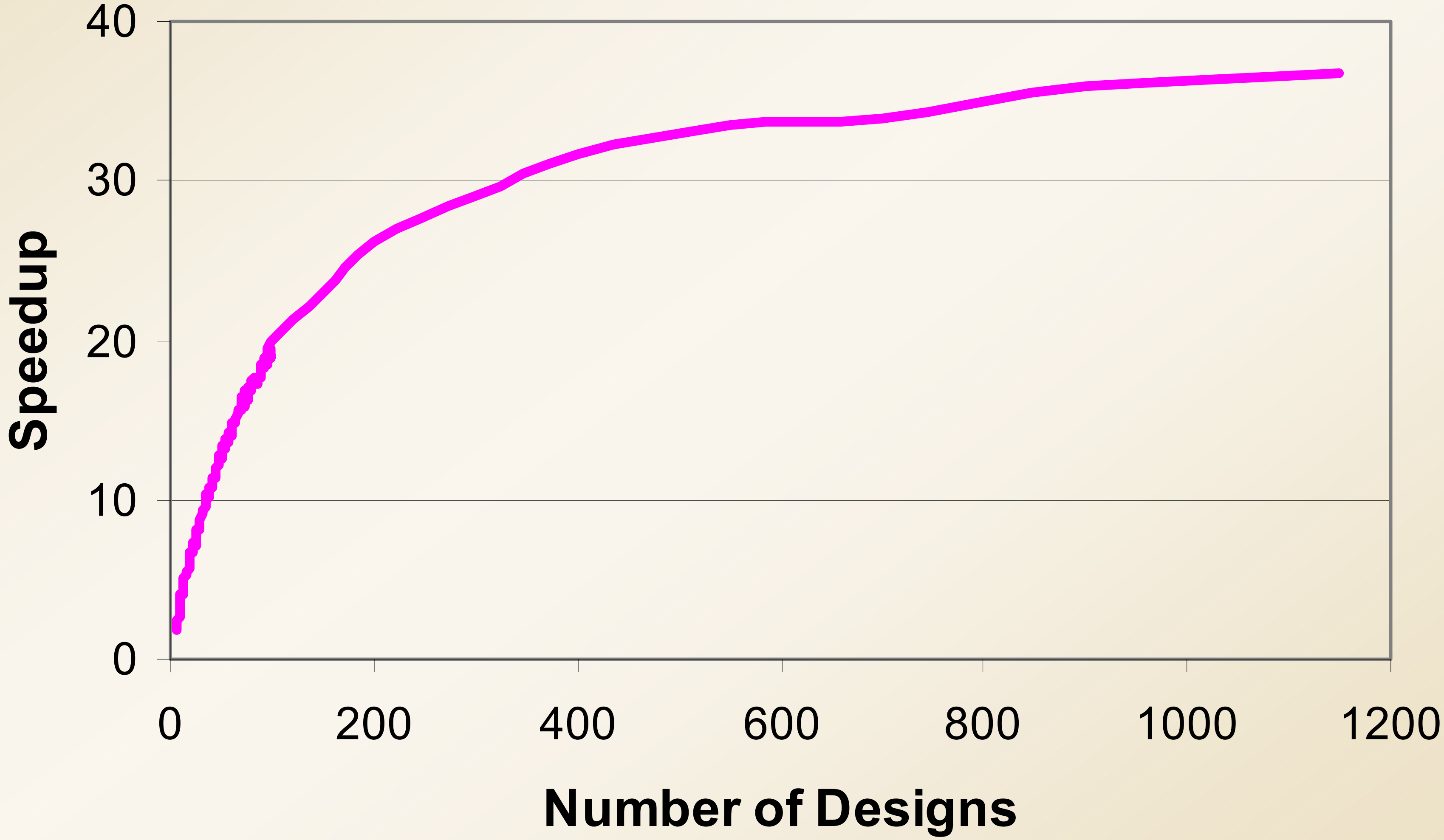


Speed-up Factors for Different Numbers of Alternatives

Number of designs, *k*	4	10	20	50	75	100
Speedup factor using OCBA	1.75	3.42	6.45	12.8	16.3	19.8

- The more alternatives a decision maker has, the higher speedup factor OCBA can achieve.

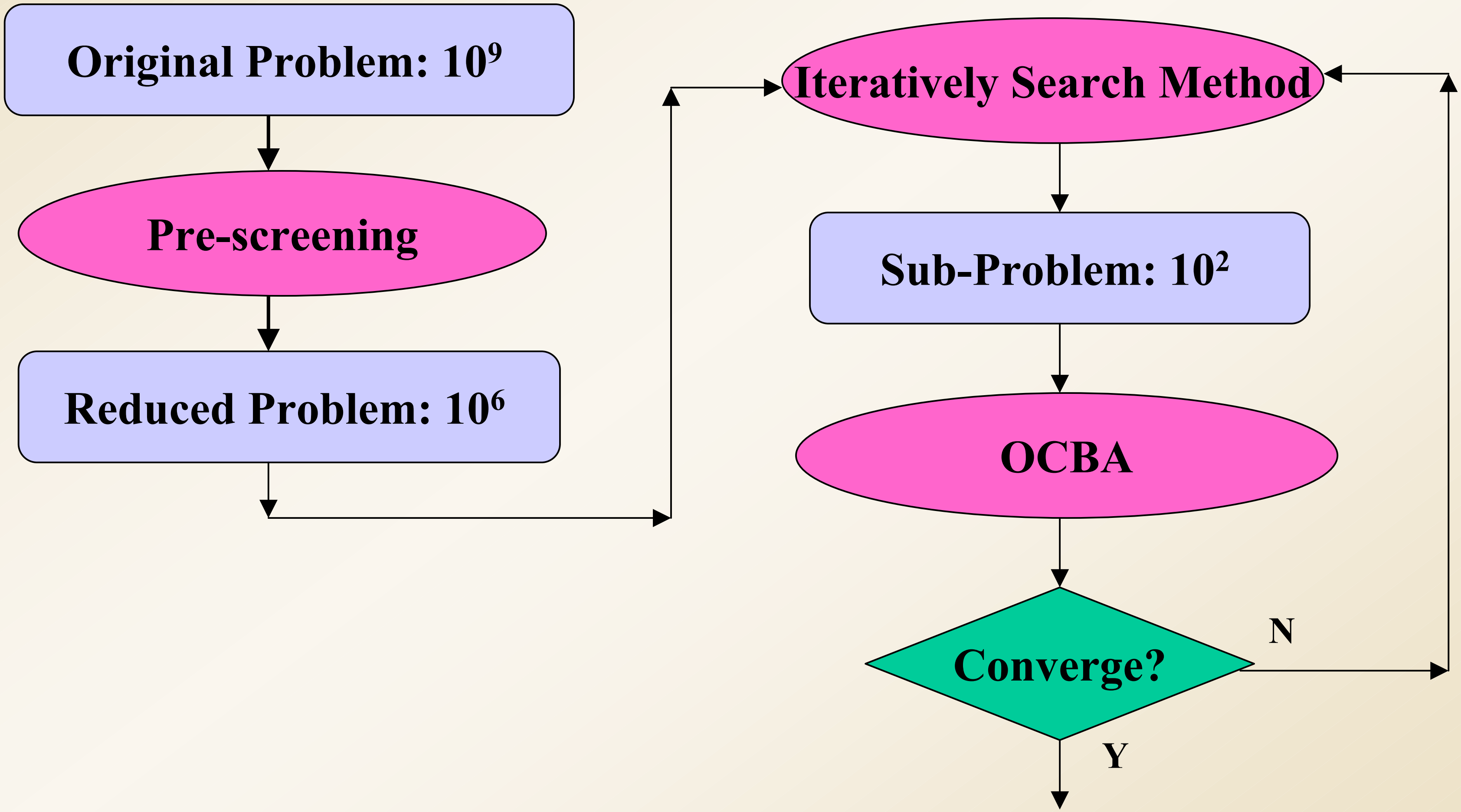
Speedup Factor vs. Design Numbers



Remarks

- OCBA is a very efficient simulation control technique for a set of design alternatives
- The more alternatives a decision maker has, the higher speedup factor OCBA can achieve
- However, there is a limit for the applicability of OCBA. For problems with a large number of design alternatives, it is still too expensive to simulate all designs.
- Integrate with other search techniques to extend OCBA to larger problems.

Use of OCBA for Much Larger Problems



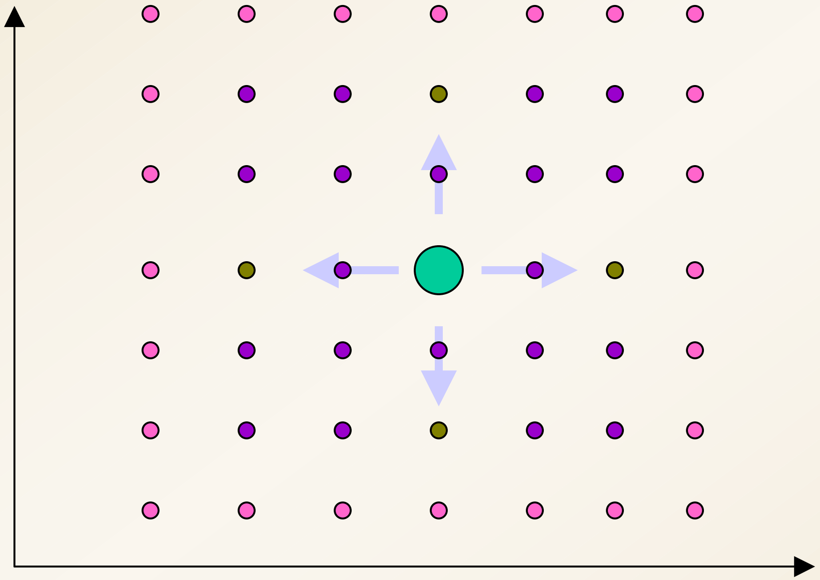
Further Development of OCBA

Extension to Continuous Design Space

Number of designs, *k*	4	10	20	50	75	100
Speedup factor using OCBA	1.75	3.42	6.45	12.8	16.3	19.8
ENAFCB(*k*)	2.29	2.93	3.10	3.90	4.59	5.05

- **ENAFCB(*k*) is defined as the equivalent number of alternatives we can simulate using the equal allocation procedure with the fixed computing budget that is needed for OCBA to simulate *k* alternatives for reaching a same *P*{CS} = 99%.**
- **With the same computation cost of simulating 4 designs, OCBA can simulate 50 designs.**

Integration with Pattern Coordinate Search



- **Collaboration with Sandia National Laboratories**
- **Sample 2 points at each coordinate**
- **With the same computation cost of simulating 4 designs, OCBA can simulate 50 designs**
  - Shall we simulate 50 designs?
  - How to sample these 50 designs?

Potential Projects

- **Integration with other search techniques**
  - Genetic Algorithm
  - Nested Partition
  - Other optimization methods
- **Further development of OCBA for the different objectives**
  - $P\{\text{CS}\}$
  - E[Opportunity Cost]
  - E[$J(X_{\text{selected design}})$]