HW#3

1.

(a) Since each item will, independently, be found with probability $1 - e^{-\mu t}$ it follows that the number found will be Poisson distribution with mean $\lambda(1 - e^{-\mu t})$. Hence, the total expected return is $R\lambda(1 - e^{-\mu t}) - Ct$.

(b) Calculus now yields that the maximizing value of t is given by

$$t = \frac{1}{\mu} \log \left(\frac{R\lambda\mu}{C} \right)$$

provided that $R\lambda\mu > C$; if the inequality is reversed then $t = 0$ is best.

2.

78. Poisson with mean 63.

3.

85. \$ 40,000 and \$$1.6 \times 10^8$.

4.

Let $X(15)$ denote the daily withdrawal. Its mean and variance are as follows.

$$E[X(15)] = 12 \cdot 15 \cdot 30 = 5400$$

$$Var[X(15)] = 12 \cdot 15 \cdot [30 \cdot 30 + 50 \cdot 50] = 612.000$$

Hence,

$$P\{X(15) \leq 6000\}$$

$$= P\left\{ \frac{X(15) - 5400}{\sqrt{612,000}} \leq \frac{600}{\sqrt{612,000}} \right\}$$

$= P\{Z \leq .767\}$ where Z is a standard normal

$= .78$ from Table 7.1 of Chapter 2.

5.

$$R(3) = R(0)\ P^3 = \begin{bmatrix} 1/4 & 1/4 & 1/2 \end{bmatrix} \begin{bmatrix} 13/36 & 11/54 & 47/108 \\ 4/9 & 4/27 & 11/27 \\ 5/12 & 2/9 & 13/36 \end{bmatrix} = \begin{bmatrix} r_0(3) & r_1(3) & r_2(3) \end{bmatrix}$$

Then $E[X_3] = 1*r_1(3) + 2*r_2(3)$

6.

$$\begin{aligned} P_{30}^2 + P_{31}^2 &= P_{31}P_{10} + P_{33}P_{11} + P_{33}P_{31} \\ &= (.2)(.5) + (.8)(0) + (.2)(0) + (.8)(.2) \\ &= .26. \end{aligned}$$

7.

(i) $\{0,1,2\}$ recurrent.

(ii) $\{0,1,2,3\}$ recurrent.

(iii) $\{0,2\}$ recurrent, $\{1\}$ transient, $\{3,4\}$ recurrent.

(iv) $\{0,1\}$ recurrent, $\{2\}$ recurrent, $\{3\}$ transient, $\{4\}$ transient.

8.

The limiting probabilities are obtained from

$r_0 = .7r_0 + .5r_1$

$r_1 = .4r_2 + .2r_3$

$r_2 = .3r_0 + .5r_1$

$r_0 + r_1 + r_2 + r_3 = 1,$

and the solution is

$$r_0 = \frac{1}{4}, \quad r_1 = \frac{3}{20}, \quad r_2 = \frac{3}{20}, \quad r_3 = \frac{9}{20}.$$

The desired result is thus

$$r_0 + r_1 = \frac{2}{5}.$$

9.

(a) $$E\left[\sum_{k=0}^{\infty} X_k | X_0 = 1\right] = \sum_{k=0}^{\infty} E[X_k | X_0 = 1] = \sum_{k=0}^{\infty} \mu^k = \frac{1}{1-\mu}.$$

(b) $$E\left[\sum_{k=0}^{\infty} X_k | X_0 = n\right] = \frac{n}{1-\mu}.$$

10.

(a) $r_0 = \frac{1}{3}$

(b) $r_0 = 1.$

(c) $r_0 = \left(\sqrt{3} - 1\right) / 2.$