HW#4 OR 442

4-1.

1. We have an M/M/1 system with
λ = 10 customers/minute μ = 12 customers/minute, and $\rho = 10/12 = 5/6$.

a. $1 - \pi_0 = 5/6$

b. $L_q = \frac{p^2}{1-p} = \frac{25/36}{1/6} = 25/6$ customers.

c. $L = \frac{\rho}{1-\rho} = \frac{5/6}{1/6}$ 5 customers
$W = L/\lambda = 5/10 = 1/2$ minute = 1/120hr.

4-2.

3. Let λ, μ, L, W and π_j refer to the original system and 2λ, 2μ, L', W' and π' refer to the new system. Since the new system has the same ρ as the old system $\pi_j = \pi_j'$, and the steady state probabilities remain unchanged. Since $L = \rho/(1-\rho)$, we will have L = L' and expected queue length is unchanged. Finally, since $W = L/\lambda$ we have that $W' = L'/2\lambda = W/2$. Thus expected waiting time for new system is half expected waiting time for old system.

4-3.

4a. $L_q = 40^2/(60-40)60 = 1.33$ customers

4b. $W = 1/(60-40) = 1/20$ hour = 3 minutes

4c. $1 - \pi_0 - \pi_1 - \pi_2 - \pi_3 = 1 - 1/3 - 2/9 - 4/27 - 8/81 = 16/81$.

4-4.

7a. λ = 125 packets per second $\mu = 1/.002 = 500$ packets per second.

$L_q = 125^2/((500)(500-125)) = .083$ packets.

7b.
$\rho = 125/500 = 1/4$
$\pi_0 = 1 - \rho = 3/4$
$\pi_1 = \rho\ \pi_0 = 3/16$
$\pi_2 = \rho\ \pi_1 = 3/64$

$P(L_q >= 2) = P(L >= 3) = 1 - \pi_0 - \pi_1 - \pi_2 = 1 - 3/4 - 3/16 - 3/64 = 1/64$