# SYST 684/IT 888: Fall 2004

**Assignment #1**
**Issued: 9/13/2004**
**Due: 9/27/2004**

1. Given the random variables x, and y of dimensions $n_x$ and $n_y$, with means $\bar{x}$ and $\bar{y}$, respectively, and with covariances $P_{xx}, P_{yy}, P_{xy}$;

   a. find the mean and covariance of the $n_z$ dimensional vector

      $z = Ax + By + c$

   b. Indicate the dimensions of A, B, and c.

2. Given $z = x + \omega$, where all the variables are n-vectors, with $\omega \sim N(0, P)$, $x \sim N(\bar{x}, P_0)$ and $x$ independent of $\omega$. Find the MAP estimator of $x$.

3. A discrete-valued parameter with prior pdf: $p(x = i) = p_i, \quad i = 1,2$ is measured with additive noise $z = x + w$ where $w \sim N(0, \boldsymbol{\sigma}^2)$

(a) Find the posterior pdf of the parameter.

(b) With $p_1 = p_2 = 0.5, \boldsymbol{\sigma} = 1, z = 3$, find its MAP estimate and the MSE conditioned on z.

(c) With $p_1 = p_2 = 0.5, \boldsymbol{\sigma} = 0.1, z = 1.2$, find its MMSE estimate and the associated variance.

4. (a) Find the maximum likelihood estimate of $\theta$ in the density $p(x) = \theta e^{-\theta x}$ for $x \geq 0$.

   (i.e., with multiple observations $x_i$, estimate $\theta$)

   (b) generate randomly 10 data points $x_i$ from the distribution $p(x)$ with $\theta$= 0.5

   (c) compute $\hat{\theta}$ from sample obtained in (b), graph and compare the estimated density function $\hat{p}(x)$ and the true density function $p(x)$.

   (d) repeat (b) and (c) with a sample of size 100, discuss the results.